

Noncommutative Effects in the Black Hole Evaporation in Two Dimensions*

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Abstract

We discuss some possible implications of a two-dimensional toy model for black hole evaporation in noncommutative field theory. While the noncommutativity we consider does not affect gravity, it can play an important role in the dynamics of massless and Hermitian scalar fields in the event horizon of a Schwarzschild black hole. We find that noncommutativity will affect the flux of outgoing particles and the nature of its UV/IR divergences. Moreover, we show that the noncommutative interaction does not affect Leahy's and Unruh's interpretation of thermal ingoing and outgoing fluxes in the black hole evaporation process. Thus, the noncommutative interaction still destroys the thermal nature of fluxes. In the process, some nonlocal implications of the noncommutativity are discussed.

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I. INTRODUCTION

Since Hawking's remarkable discovering in 1974 [1], that the black hole can emit radiation (for previous attempts see, for instance, Refs. [2]) this gives rise to an intensive work on quantum field theory on curved spaces (for some reviews on Hawking radiation and quantum fields on curved space, see for instance [3, 4, 5, 6, 7, 8]. In his paper Hawking considered a non-self-interacting massless Hermitian scalar field coupled to a background (non-dynamical) classical gravitational field given by the Schwarzschild spacetime. The simplest model for a matter field is given by a spin zero neutral scalar particle with mass m described by a Klein-Gordon equation, propagating on the extended Schwarzschild spacetime.

In the absence of interacting fields, the Schwarzschild black hole of mass M , will emit thermal particles with a temperature $T_H = 1/8\pi M$ [1]. The final state of this radiation will be that of an outgoing particle flux at the temperature T_H . The black hole in equilibrium with a thermal bath at temperature T_H was investigated in Ref. [9]. There it was found that the outgoing flux of radiation remains thermal due to detailed balance arguments.

The question if the thermal nature of the radiation remains unaltered when the self-interactions of matter fields surrounding the black hole are taken into account was discussed in [10, 11] (for previous results, see [12]). To be more precise, they considered a toy model based in a massless real scalar field Φ with a self-interacting term of the form: $\lambda\Phi^4$, with λ being the coupling constant (this model was introduced and studied in more detail in Ref. [12]). This field is defined in the two-dimensional space-time defined by the coordinates (r, t) of a four-dimensional Schwarzschild black hole. Thus the background metric is not by itself solution of a two-dimensional Einstein theory (which is non-dynamical). In [10, 11] it was found that the this interacting model in the black hole is equivalent to a model of the same scalar field defined in flat space-time but with a spatially varying parameter λ . In fact, they parallelly perform both calculations and interpret them in physical terms.

The outgoing and ingoing fluxes can interact with each other as two separated thermal baths (generically) at different temperatures. For a massless field in flat two-dimensional space-time, outgoing and ingoing thermal fluxes would remain in equilibrium with themselves even if they are at different temperatures. The reason of this is that the spatial symmetries forbid the interaction between the two fluxes, remaining thermal. Moreover, the presence of the black hole breaks translation invariance (and therefore momentum conser-

vation is violated). Then the interaction between the fluxes is possible and this interaction can be regarded as a varying λ in the spatial direction where the linear momentum conservation was broken (in this case the coordinate is r , so we have $\lambda(r)$). In this respect in Ref. [13] it was investigated the particle emission for a massless Thirring model in a curved space. In this case, the outgoing flux remained thermal due to conformal invariance of the two-dimensional model.

In this spirit, the introduction of interactions of the type $\lambda\Phi^4$ which does not respect conformal invariance would lead to a deviation from the thermal radiation. In fact this was the case, and in [10, 11] it was found that if the ingoing state is the vacuum state, the thermal outgoing flux will be destroyed by the interaction. But, if both thermal fluxes, coupled by the interaction, have the same temperature $T = T'$, then detailed balance will maintains the outgoing flux thermal. In the general case that both thermal fluxes have different temperatures $T \neq T'$, then the interaction will destroys the thermal nature of both fluxes. In the present paper we will study some effects over this system of coupled thermal fluxes by a noncommutative interaction.

In the computation of the perturbative corrections to the flux, in [10, 11] a diagrammatic set of rules to compute these corrections was given. The computations of the amplitude have virulent divergences which do not avoid giving a physical interpretation.

On the other hand, the idea of noncommutative spacetime coordinates is old and has been in the literature during many years [14]. Moreover, the noncommutative field theories have been extensively explored recently mainly in the context of string theory [15] and quantum field theories (for some recent reviews, see [16]). In particular, in these theories there are new and intriguing effects such as the UV/IR mixing (at one loop) [17] and also two-loops effects [18]. Their application to gravitational theories has been the subject of recent renewed interest (see for instance, [19]). However noncommutative field theory in curved spaces has been not explored exhaustively in the literature. This will be the aim of this paper and we will explore this subject by studying the concrete example of the Hawking's evaporation of a two-dimensional model of a black hole worked out in Refs. [10, 11].

Recently, some cosmological models with both gravity and matter being noncommutative and dynamical are described in the context of the famous Connes-Lott model in Ref. [20]. Another discussion of the noncommutative matter propagating in a noncommutative dynamical (but linearized) spacetime was done in [21]. Here gravitational anomalies in

several dimensions were computed, including the computation of anomaly cancellation in noncommutative supergravity in ten dimensions.

More recently some work has been done in the context in which the noncommutativity does not affect gravity, and only affects to the matter in a classical (and commutative) dynamical spacetime. However, if originally only the matter is noncommutative, then the noncommutativity will have some influence in the proper dynamics of the spacetime. An example of this is in Ref. [22], where some cosmological implications of noncommutative matter in a fixed and (commutative) background cosmological metric were studied. It was found that noncommutativity might cause inflation induced fluctuations to become non-Gaussian and anisotropic in such a way that they would modify the short distance dispersion relations. In the present paper we will adopt this same philosophy such that noncommutativity does not affect gravity but still affect the dynamics of massless real scalar field and its self-interaction of the form: $\lambda\Phi \star \Phi \star \Phi \star \Phi$. We shall see that even for this approximation, the system will receive non-trivial noncommutative corrections to the outgoing thermal flux, its thermally due this interaction and the structure of its divergences. Other works in this context deal with the noncommutative description of the coupling of scalar fields to gravity. For instance, in Ref. [23] the Moyal product is extended to describe of the coupling of a massive scalar field coupled to gravity.

Recently, there has been several attempts to understand Hawking's radiation in a noncommutative spacetime. Perhaps one of the first attempts was given in Ref. [24]. In this paper the effects of the noncommutative spacetime are introduced by a direct modification of the radiation field through a q -deformation of its oscillator Heisenberg algebra. Also in this case the metric is unchanged (i.e. it remains commutative). These assumptions modify the expectation values of the oscillator number operator and therefore the radiation spectrum of a Schwarzschild black hole are also modified. It predicts also an IR cut-off of the spectrum. This model is very different from that proposed here, as we will consider self-interacting matter fields in a curved background.

Some of others recent works in this direction are [25], in where the effects of a noncommutative spacetime are encoded in some Schwarzschild black hole properties as horizon, area spectrum and Hawking temperature T_H . This is done by modifying the spacetime metric into a proposed noncommutative generalization. The deviations of these properties from the usual ones depends of the noncommutativity parameter Θ . Moreover Refs. [26] investigated,

by very different methods, the behavior of a radiating Schwarzschild black hole toy model in two dimensions. The existence of a minimal non-zero mass to which the black hole can be shrink and a finite maximum temperature that the black hole can reach was shown. Here the absence of any curvature singularity was also discussed.

In the context of string theory, the noncommutative field theories in curved spaces can be derived as an effective theory of gravity in a curved D3-brane in the presence of an electromagnetic field [27]. This give rise to gravity coupled noncommutative Maxwell field and it was studied as the non-linear terms does modify Hawking's radiation.

Very recently a toy model for the black hole in a noncommutative space in two space dimensions was constructed in [28]. In almost the same direction in Ref. [29], they constructed a model where the event horizon of a black hole is a fuzzy sphere and in the classical limit it is found that the event horizon looks locally like a noncommutative plane with a noncommutative parameter represented by the Planck length. Moreover, the analysis of the quasinormal modes in a noncommutative black hole was studied in Ref. [30]. In this work some of the results of the last reference of [26] were used in order to study the asymptotic quasinormal models of the scalar perturbation of the noncommutative geometry of a Schwarzschild black hole in $(3 + 1)$ dimensions. Here a explicit dependence of the T_H in terms of the noncommutativity parameter is found. Finally, the noncommutative Schwarzschild black hole has been studied in the context of black hole/cosmology duality and the noncommutative minisuperspace in [31].

In this paper we will reexamine Leahy and Unruh approach [10, 11]. Differing from the approaches mentioned above, in the present paper we introduce the computation of the noncommutative effects in Hawking's radiation in the context of noncommutative interacting quantum field theory. In order to keep the work as self-contained as possible, in Section II we give a brief review of the effects of the $\lambda\Phi^4$ interaction on the coupled thermal fluxes of outgoing and ingoing particles in a black hole (evaporation) in two dimensions, following Refs. [10, 11]. In Sections III, IV and V, we proceed to explain how to “deform” this model. We propose promoting the usual product between the mode functions in the interaction of the field Φ , which give rise to a noncommutative interaction $\lambda\Phi \star \Phi \star \Phi \star \Phi$ (see [16]) to investigate the effect produced by this modification in the outgoing particle flux. In Section VI we show that noncommutativity affect both thermal fluxes and the thermally of these fluxes in the black hole evaporation process. Finally, in Section VII we give our final remarks.

II. OVERVIEW OF THE MODEL OF BLACK HOLE EVAPORATION IN TWO-DIMENSIONS

In this section we review some important aspects of the influence of the self-interacting term of a matter field Φ over the Hawking's radiation in a two-dimensional model. To be more precise we will follow the Leahy and Unruh Ref. [10], where a self-interacting term for a Hermitian and massless scalar field of the form $\lambda\Phi^4$ in two dimensions and coupled to the gravitational field given by the (r, t) part of the Schwarzschild metric was introduced and the effects of this term over the ingoing and outgoing thermal fluxes were computed. Our aim is not to provide an extensive review but briefly recall some of their relevant properties. We will adopt the notation and conventions from Leahy and Unruh's paper. For instance we use natural units such that $G = \hbar = c = k = 1$.

As we have stated before, scalar massless fields Φ satisfy the wave equation

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi\right)=0, \quad (1)$$

where $g_{\mu\nu}$ is the two-dimensional part of the spacetime metric of a four-dimensional Schwarzschild black hole. This is given by

$$\begin{aligned} ds^2 &= \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1}dr^2 \\ &= \left(1 - \frac{2M}{r}\right)dudv, \end{aligned} \quad (2)$$

where the null coordinates u and v are defined as $u = t - r^*$ and $v = t + r^*$. Here r^* is the tortoise coordinate

$$r^* = r + 2M \ln\left(\frac{r}{2M} - 1\right). \quad (3)$$

The main reason for studying such a two-dimensional system is because the scalar field solutions are much simpler in this case.

We will focus on a system composed of two thermal fluxes: one of them consisting of outgoing particles and the other one of ingoing particles at temperatures β and β' respectively, coupled by the interacting term $\lambda\Phi^4$.

Now let's consider the solution of Eq. (1). We can express these fields as

$$\Phi(u, v) = \Phi_O(u) + \Phi_I(v), \quad (4)$$

where Φ_O and Φ_I are fields depending on coordinates u and v respectively. Subindexes stands for outgoing fields (O) or ingoing ones (I) as we will see. In order to quantize the field, we expand these solutions into normal modes, using appropriate creation and annihilation operators. The simplest expansion for ingoing and outgoing modes respectively is given by

$$\chi_\omega = N \frac{e^{-i\omega v}}{(2|\omega|)^{1/2}}, \quad \psi_\omega = N \frac{e^{-i\omega u}}{(2|\omega|)^{1/2}}, \quad (5)$$

where N is a normalization factor independent of the frequency.

Thus the components of Eq. (4) can be written as

$$\Phi_I = \sum_{\omega>0} (b_\omega \chi_\omega + b_\omega^\dagger \chi_\omega^*), \quad (6)$$

$$\Phi_O = \sum_{\omega>0} (C_\omega \psi_\omega + C_\omega^\dagger \psi_\omega^*), \quad (7)$$

where b_ω , b_ω^\dagger and C_ω , C_ω^\dagger are harmonic oscillator operators for ingoing and outgoing modes.

Now we will define the vacuum state for ingoing modes as the state $|s\rangle$ for which

$$b_\omega |s\rangle = 0, \quad (\omega > 0). \quad (8)$$

The corresponding state $|s'\rangle$ defined with the C_ω operator represent the vacuum state for outgoing modes, i.e.,

$$C_\omega |s'\rangle = 0, \quad (\omega > 0). \quad (9)$$

As usual, $|s\rangle$ and $|s'\rangle$ are related by a *Bogoliubov* transformation. In these terms the thermal flux of outgoing particles from the black hole with frequency ω at infinity corresponds to expectation value of the number operator $C_\omega^\dagger C_\omega$, i.e.,

$$\frac{dF}{d\omega}(\omega) = \frac{1}{\pi} \text{tr} \left(\rho C_\omega^\dagger C_\omega \right), \quad (10)$$

where ρ is the density matrix constructed as follows

$$\rho = \rho_O \otimes \rho_I,$$

with

$$\rho_O = |0\rangle\langle 0|, \quad (11)$$

$$\rho_I = \bigotimes_{\omega} \sum_{n_{\omega}} e^{-n_{\omega}\omega\beta'} |n_{\omega}\rangle_I \langle n_{\omega}|_I. \quad (12)$$

Here $|n_{\omega}\rangle_I$ is the state with n quanta of the ingoing mode with energy ω at temperature β' .

Now we shall examine what happens when one introduces a self-interacting term for the scalar field Φ . We are particularly interested in the expectation value of the outgoing flux at infinity. Using the interaction picture the density matrix will evolve [32] through a S -matrix such that

$$\rho(t) = S(t)\rho(0)S^{\dagger}(t), \quad (13)$$

where $\rho(0)$ is the initial density matrix. $S(t)$ will be defined in the next section.

III. NONCOMMUTATIVE DEFORMATION OF BLACK HOLE EVAPORATION IN TWO DIMENSIONS

In this section we introduce a noncommutative interaction in the two-dimensional background model of the type $\lambda\Phi_{\star}^4 \equiv \lambda\Phi \star \Phi \star \Phi \star \Phi$. The meaning of the star operation \star will be also explained in this section. The coordinates u and v depend on the noncommutative canonical coordinates (r, t) as usual: $u = t - r^*$ and $v = t + r^*$ where r^* was defined previously. Then, the fields Φ depend on the noncommuting coordinates $x^{\mu} = (r, t)$, i.e. $[x^{\mu}, x^{\nu}] = i\Theta^{\mu\nu}$. Thus we promote all products of the normal modes functions to star products [16]. It is natural then to define the Moyal product:

$$(\Phi_1 \star \Phi_2)(x) \equiv \left[e^{\frac{i}{2}\Theta^{\mu\nu}\partial_{\xi\mu}\partial_{\eta\nu}} \Phi_1(x + \xi)\Phi_2(x + \eta) \right]_{\xi=\eta=0}, \quad (14)$$

where $\Theta^{\mu\nu} = \Theta\varepsilon^{\mu\nu}$ is the matrix determined by the noncommutative parameter Θ .

Now we introduce the noncommutative interaction, modifying the equations for $S(t)$ given in [10] as follows

$$S^{\star}(t) = \text{T exp} \left[-i \int^t H_I^{\star}(t') dt' \right], \quad (15)$$

where the noncommutative Hamiltonian $H_I^{\star}(t)$ is given by

$$\begin{aligned}
H_I^*(t) &= \int \frac{\lambda}{4} \Phi_1 \star \Phi_2 \star \Phi_3 \star \Phi_4 \, dr \\
&= \frac{\lambda}{4} \int dr \left(\Phi(x_1) \star \Phi(x_2) \right) \left(\Phi(x_3) \star \Phi(x_4) \right) \\
&= \frac{\lambda}{4} \int dr e^{-\frac{i}{2} \overleftarrow{\partial}_1 \Theta^{12} \overrightarrow{\partial}_2} e^{-\frac{i}{2} \overleftarrow{\partial}_3 \Theta^{34} \overrightarrow{\partial}_4} \Phi(x_1) \Phi(x_2) \Phi(x_3) \Phi(x_4).
\end{aligned} \tag{16}$$

Interaction term (figure 1) should be symmetric under the cyclic permutation of any pair of fields. Thus we have to symmetrize [16, 18] the last expression to obtain

$$\frac{\lambda}{12} \int dr F(\Theta) \cdot \Phi(x_1) \Phi(x_2) \Phi(x_3) \Phi(x_4), \tag{17}$$

where

$$F(\Theta) = \cos \frac{\partial_1 \Theta \partial_2}{2} \cos \frac{\partial_3 \Theta \partial_4}{2} + \cos \frac{\partial_1 \Theta \partial_3}{2} \cos \frac{\partial_2 \Theta \partial_4}{2} + \cos \frac{\partial_1 \Theta \partial_4}{2} \cos \frac{\partial_2 \Theta \partial_3}{2}. \tag{18}$$

Here we introduced the notation $\partial_i \Theta \partial_j \equiv \overleftarrow{\partial}_i \Theta^{ij} \overrightarrow{\partial}_j$. Then at the level of the interaction, noncommutative corrections only introduces a factor $F(\Theta)$ given by the Cosine terms. This factor is precisely what we need to introduce in the “fat” vertex of the interaction (see figure 1).

At the same time, we see from Eq. (15), that the noncommutative version to the interaction Hamiltonian is carried out to the evolution of the density matrix and therefore to the flux of outgoing particles by

$$\begin{aligned}
\frac{dF^*}{d\omega}(\omega) &= \frac{1}{\pi} \text{tr} \left(\rho(t) \star C_\omega^\dagger C_\omega \right) \\
&= \frac{1}{\pi} \text{tr} \left(S^*(t) \rho(0) S^{\dagger}(t) C_\omega^\dagger C_\omega \right),
\end{aligned} \tag{19}$$

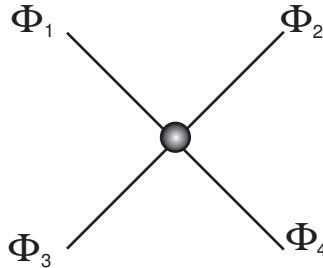


FIG. 1: Diagram of the noncommutative interaction $\lambda \Phi \star \Phi \star \Phi \star \Phi$ represented by a fat vertex.

where $S^*(t) = 1 + S_1^*(t) + S_2^*(t) + \dots$. Due the fact that $\rho(0)$ and C_ω are independent of local coordinates r and t and $S^*(t)$ depending only on t , the Moyal product is not apparent in Eq. (19).

Computing now $S^*(t)$ at the second order in the coupling constant λ and demanding $S^*S^{\dagger} = 1$, we have

$$\begin{aligned} \left. \frac{dF^*}{d\omega} \right|_2(\omega) &= \frac{1}{\pi} \sum_{\alpha} p_{\alpha} \left\langle \alpha \left| S_1^{\dagger} [C_{\omega}^{\dagger} C_{\omega}, S_1^*] \right| \alpha \right\rangle \\ &= \frac{1}{\pi} \sum_{\alpha\beta} p_{\alpha} (n_{\omega\beta} - n_{\omega\alpha}) \left| \left\langle \beta \left| S_1^* \right| \alpha \right\rangle \right|^2, \end{aligned} \quad (20)$$

where $\{|\alpha\rangle\}, \{|\beta\rangle\}$ are complete and orthogonal sets of states for the field Φ . They are selected such that they will be eigenstates of the number operator for each one of the modes of Φ . Here p_{α} stands for the thermal probability functions for the states $|\alpha\rangle$ in the density matrix $\rho(0)$. Also $n_{\omega\alpha}$ and $n_{\omega\beta}$ are the number of outgoing quanta of energy ω at the states $|\alpha\rangle$ and $|\beta\rangle$, respectively. The expression: $|\langle\beta|S_1^*|\alpha\rangle|^2$ represents the probability transition of the system that starting at state $|\alpha\rangle$ evolves until the final state $|\beta\rangle$ under the noncommutative interaction and finally, $n_{\omega\beta} - n_{\omega\alpha} \equiv \Delta n_{\omega}$ is the difference in the number of outgoing particles of energy ω during the process.

In the derivation of Eq. (20) we have used the fact

$$S_1^* + S_1^{\dagger} = 0, \quad (21)$$

$$S_1^* S_1^{\dagger} + S_2^{\dagger} + S_2^* = 0. \quad (22)$$

Here S_1^* depends only on the coordinate t , which implies that the Moyal product reduces to the product of functions. Moreover if one take into account Eq. (17) for S_1^* and S_1^{\dagger} , we will have that the noncommutative corrections to the flux (Eq. (20)) will start at the second order in the noncommutativity parameter Θ . Thus, noncommutative corrections come from the noncommutative interaction term $\lambda\Phi_{*}^4$. Scalar fields can be expanded in orthogonal modes whose basis is given by plane waves in Eqs. (5). The exponential dependence of the modes implies that the Moyal product between two fields has infinite number of terms. Thus an exact computation is not possible and this is the reason why we will adopt here

the perturbative expansion in $\Theta \ll 1$. In order to be concrete we will compute lowest noncommutative corrections which will be quadratic in Θ .

Now we proceed to perform the noncommutative corrections to the $S_1^*(t)$. Then we have

$$S_1^*(t) = -\frac{i}{4} \int^t \lambda \left(\Phi_1 \star \Phi_2 \star \Phi_3 \star \Phi_4 + \text{permutations} \right) dr' dt', \quad (23)$$

here “permutations” stands for the addition of all permutations leading to the form of Eq. (17) which is manifestly symmetric under the interchange of any pair of fields (see Eq. (27) below).

In the section IV we will compute the first non-vanishing noncommutative correction which is at the second order in the expansion of the Moyal products of the interaction Hamiltonian H_I^* . Thus we can write it as

$$H_I^* = H_I + H_I^{NC}[\Theta^2] + \mathcal{O}[\Theta^4], \quad (24)$$

where $H_I^{NC}[\Theta^2]$ will be given by Eq. (31) (see below). If we substitute this last expression in S_1^* we get

$$S_1^* = S_1 + S_1^{NC}[\Theta^2] + \mathcal{O}[\Theta^4], \quad (25)$$

with S_1 being the usual (commutative) action worked out in Ref. [10, 11]. Here $S_1^{NC}[\Theta^2]$ is given by

$$S_1^{NC}[\Theta^2] = -i \int H_I^{NC}[\Theta^2] dt. \quad (26)$$

In the next section we are going to consider the corrections to the flux of outgoing particles of the black hole background.

IV. NONCOMMUTATIVE CORRECTION TO THE INTERACTION HAMILTONIAN FOR LARGE $1/\Theta$

We have mentioned that the noncommutative correction of the interaction Hamiltonian at the first order in the noncommutativity parameter Θ can be computed and it vanishes. It is easy to prove that the sum of the first order contributions vanishes. To see this take the interaction $\lambda \Phi_\star^4$ with the symmetrized products of the fields as follows:

$$\begin{aligned}
\int dr \lambda \Phi_\star^4 = \frac{\lambda}{6} \int dr \bigg(& \Phi_1 \star \Phi_2 \cdot \Phi_3 \star \Phi_4 + \Phi_1 \star \Phi_2 \cdot \Phi_4 \star \Phi_3 + \Phi_2 \star \Phi_1 \cdot \Phi_3 \star \Phi_4 \\
& + \Phi_2 \star \Phi_1 \cdot \Phi_4 \star \Phi_3 + \Phi_1 \star \Phi_3 \cdot \Phi_2 \star \Phi_4 + \Phi_1 \star \Phi_3 \cdot \Phi_4 \star \Phi_2 \\
& + \Phi_3 \star \Phi_1 \cdot \Phi_2 \star \Phi_4 + \Phi_3 \star \Phi_1 \cdot \Phi_4 \star \Phi_2 + \Phi_1 \star \Phi_4 \cdot \Phi_2 \star \Phi_3 \\
& + \Phi_1 \star \Phi_4 \cdot \Phi_3 \star \Phi_2 + \Phi_4 \star \Phi_1 \cdot \Phi_2 \star \Phi_3 + \Phi_4 \star \Phi_1 \cdot \Phi_3 \star \Phi_2 \bigg). \quad (27)
\end{aligned}$$

Of course, this expression is equivalent to Eq. (17).

One knows that the Moyal product depends on $\Theta^{\mu\nu}$ which is anti-symmetric and therefore terms like the first term $\Phi_1 \star \Phi_2 \cdot \Phi_3 \star \Phi_4$ and the second term $\Phi_1 \star \Phi_2 \cdot \Phi_4 \star \Phi_3$ are the negative one of each other at the first order in Θ , and therefore they cancel. Something similar happens for the other pairs of terms and then we conclude that the noncommutative correction to the flux of particles (see Eq. (18)) will have the first non-vanishing correction at the second order in Θ .

The noncommutative amplitude $\langle \beta | S_1^\star | \alpha \rangle$ necessary to find the outgoing flux of radiation can be computed by using a diagrammatic representation. The introduction of the respective rules will be the aim of the following section. To compute the matrix entries of S_1 and S_1^{NC} , we use the perturbative techniques of noncommutative field theories. From Eq. (17) we expand the cosine functions and we can rewrite one of the terms in the noncommutative factor in the following form

$$\cos\left(\frac{\partial_1 \Theta \partial_2}{2}\right) \cos\left(\frac{\partial_3 \Theta \partial_4}{2}\right) = 1 - \frac{1}{2!} \left[\left(\frac{\partial_1 \Theta \partial_2}{2}\right)^2 + \left(\frac{\partial_3 \Theta \partial_4}{2}\right)^2 \right] + \mathcal{O}[\Theta^4]. \quad (28)$$

The terms inside the bracket can be expanded as

$$\begin{aligned}
(\partial_1 \Theta \partial_2)^2 &\equiv (\overleftarrow{\partial}_{\mu 1} \Theta^{\mu\nu} \overrightarrow{\partial}_{\nu 2})^2 \\
&= \Theta^2 (\overleftarrow{\partial}_1 \overrightarrow{\partial}_2 - \overleftarrow{\partial}_2 \overrightarrow{\partial}_1)^2, \quad (29)
\end{aligned}$$

with similar results for $(\partial_3 \Theta \partial_4)^2$. Substituting these expressions into Eq. (17) and computing the first non-vanishing contribution we have

$$-\frac{\Theta^2}{8} \Phi(x_1) [\overleftarrow{\partial}_r^2 \overrightarrow{\partial}_t^2 - 2 \overleftarrow{\partial}_r \overleftarrow{\partial}_t \overrightarrow{\partial}_r \overrightarrow{\partial}_t + \overleftarrow{\partial}_t^2 \overrightarrow{\partial}_r^2] \Phi(x_2) \Phi(x_3) \Phi(x_4). \quad (30)$$

Now using the modes expressions (5) in the computation of the noncommutative correction (30), after some laborious but easy computations, we finally get

$$H_I^{NC}(\Theta^2) = - \int dr \frac{\lambda \Theta^2}{96} \Phi(x_1) [\overleftarrow{\partial}_r^2 \overrightarrow{\partial}_t^2 - 2 \overleftarrow{\partial}_r \overleftarrow{\partial}_t \overrightarrow{\partial}_r \overrightarrow{\partial}_t + \overleftarrow{\partial}_t^2 \overrightarrow{\partial}_r^2] \Phi(x_2) \Phi(x_3) \Phi(x_4) + \text{permut.}$$

$$\begin{aligned}
&= - \int dr \frac{\lambda \Theta^2}{24} \left(1 - \frac{2M}{r}\right)^{-2} \sum_{\omega_1, \omega_2} \left\{ \omega_1^2 \Phi_I(x_1) \omega_2^2 \Phi_O(x_2) + \omega_1^2 \Phi_O(x_1) \omega_2^2 \Phi_I(x_2) \right. \\
&+ \left. i \frac{M}{r^2} \left[\omega_1 \left(b_1^\dagger \chi_1^* + C_1 \psi_1 \right) \left(\omega_2^2 \Phi(x_2) \right) + \left(\omega_1^2 \Phi(x_1) \right) \omega_2 \left(b_2^\dagger \chi_2^* + C_2 \psi_2 \right) \right] \right\} \Phi(x_3) \Phi(x_4) + \text{permut.}
\end{aligned} \tag{31}$$

Proceeding in a similar manner with the term $\Phi(x_3)(\partial_3 \Theta \partial_4 / 2)^2 \Phi(x_4)$ we obtain the same equation now with subindexes $1 \leftrightarrow 3$ and $2 \leftrightarrow 4$ interchanged. Finally we have to add the contributions coming from the remaining terms $\cos(\partial_1 \Theta \partial_3 / 2) \cos(\partial_2 \Theta \partial_4 / 2) + \cos(\partial_1 \Theta \partial_4 / 2) \cos(\partial_2 \Theta \partial_3 / 2)$. This will give the first non-vanishing noncommutative correction of H_I^* .

V. PLANAR DIAGRAMMATIC RULES OF THE NONCOMMUTATIVE FLUX

Our final aim is the computation of the noncommutative corrections to the flux of outgoing particles. To compute such corrections we first need to find the elements of the S_1^* -matrix i.e., the amplitude: $\langle \beta | S_1^* | \alpha \rangle$. To find it will be very useful to give some diagrammatic rules [10, 11], which will be slightly modified in the noncommutative field theory. Every tree diagram consist of a vertex with four lines converging at it. At one-loop we will have the usual self-energy diagram. These diagrams will have a one-to-one correspondence with the factors conforming $\langle \beta | S_1^* | \alpha \rangle$.

The interaction vertex of the noncommutative quartic interaction involves the computation of planar and non-planar diagrams at each order in perturbation theory in λ . For tree diagrams we will have planar diagrams since non-planar diagrams arise when propagator lines cross over other propagator lines or over external lines. In a perturbative expansion in Θ each planar diagram give rise to an infinite number of diagrams involving derivative of modes as external lines (see Fig. 3). In this paper we will be interested mainly in tree diagrams and therefore all diagrams we will consider are planar. Non-planar diagrams are important for loop-diagrams. We will show that even in this case, the effect of a curved background gives rise to non-trivial corrections to the flux of Hawking's radiation. We left the corrections due non-planar diagrams for a future work.

Then the rules are the following:

1. The interaction term $\lambda\Phi_\star^4$ inside $\langle\beta|S_1^\star|\alpha\rangle$ (Eq. (20)) is written under the normal ordering and the symmetrized condition (see Eq. (17)).
2. For each term $C_\omega\psi_\omega$ draw a line with right-directed arrow pointing to the noncommutative vertex, which represents an outgoing particle in the initial state.
3. For each $C_\omega^\dagger\psi_\omega^\star$ draw a line with a right-directed arrow pointing away from the noncommutative vertex and this will represent an outgoing particle in the final state.
4. For each term $b_\omega\chi_\omega$ one can draw a line with a left-directed arrow pointing to the noncommutative vertex. This represents an ingoing particle in the initial state.
5. A $b_\omega^\dagger\chi_\omega^\star$ term indicates that one can draw a line with a left-directed arrow pointing away from the noncommutative vertex. This represents an ingoing particle in the final state.
6. For the terms of the form: $\chi_{\omega_1}^\star \star \chi_{\omega_2}\delta_{\omega_1,\omega_2}$ or $\psi_{\omega_1}^\star \star \psi_{\omega_2}\delta_{\omega_1,\omega_2}$ obtained from the normal ordering of the field Φ , one can draw a loop attached to the noncommutative vertex.

Now, the rules to compute the amplitude and therefore the contribution to the flux $\frac{dF^\star}{d\omega}|_2$ are given by:

- The matrix elements are formed using the states $\langle\beta|$ and $|\alpha\rangle$ of the four operators b 's and C 's in the normal-ordered form associated with the four lines.
- Multiply by $(i/4)\int drdt$ the four mode functions associated with the four lines of a diagram.
- Multiply by an integer factor which is the number of occurrences of this term in the normal ordering of $\langle\beta|\lambda\Phi_\star^4|\alpha\rangle$. In the evaluation of it there is present the symmetry factor $\left[\cos \frac{\partial_1\Theta\partial_2}{2} \cos \frac{\partial_3\Theta\partial_4}{2} + \cos \frac{\partial_1\Theta\partial_3}{2} \cos \frac{\partial_2\Theta\partial_4}{2} + \cos \frac{\partial_1\Theta\partial_4}{2} \cos \frac{\partial_2\Theta\partial_3}{2} \right]$.
- Take the absolute square of the product from the three above steps.
- Multiply the result of the above step by $\Delta n_\omega p_\alpha/\pi$ and take the sum over all states $|\alpha\rangle$ and $|\beta\rangle$. Here Δn_ω is the number of lines for energy ω with right pointing arrows lying to the right of the vertex minus the number of lines lying to the left of the vertex.

- Finally, sum over all of the different ω_i which are not equal to ω .

It is worthwhile to remark that we will consider only the contribution of diagrams for which $|\Delta n_\omega| = 1$. The reason is as follows: the integrals of the mode functions contains terms of the form

$$\int e^{-i(\omega - \omega_1 - \omega_2 - \omega_3)t} dt \approx \delta(\omega - \omega_1 - \omega_2 - \omega_3). \quad (32)$$

This gives the energy conservation principle, which impose the restriction $|\Delta n_\omega|$ to take the values 0, 1, 2 or 3. The case $|\Delta n_\omega| = 4$ obviously violates the energy conservation unless that $\omega = 0$. $|\Delta n_\omega| = 0$ does not contribute to the particle flux and $|\Delta n_\omega| = 2$ or 3 already have been evaluated in Ref. [10, 11] and it was found that their contributions do not shed more light than the case discussed here.

All diagrams with $\Delta n_\omega = +1$ represent the inverse of the processes with $\Delta n_\omega = -1$. In fact, the relation between both diagrams is very simple because we can obtain one from the other by just making a reflection with respect to the vertical axis and inverting the orientation of all arrows.

It is possible to see that, the only diagrams with nontrivial contributions to the non-commutative outgoing flux $\left. \frac{dF^*}{d\omega} \right|_2(\omega)$ are those listed in the figure 2, which can be evaluated explicitly using the expansions (20).

VI. COMPUTATION OF THE NONCOMMUTATIVE OUTGOING RADIATION FLUX

In this section we will compute the noncommutative corrections to the flux of outgoing particles at the second order in the coupling constant λ [10, 11] and also at the second order in the noncommutativity parameter Θ .

A. Relation to the Commutative Case

In order to be concrete we will concentrate in the diagram number 1 of the figure 2 (and its reflection) and we will follow the diagrammatic rules described in the previous section, the amplitude can be computed in the following form:

$$\frac{2(12)^2}{16\pi} \sum_{\omega_1, \omega_2, \omega_3} \sum_{\alpha, \beta} p_\alpha \left(\left| \left\langle \beta \left| C_\omega^\dagger C_1 C_2 b_3 \right| \alpha \right\rangle \right|^2 - \left| \left\langle \beta \left| C_1^\dagger C_2^\dagger b_3^\dagger C_\omega \right| \alpha \right\rangle \right|^2 \right)$$

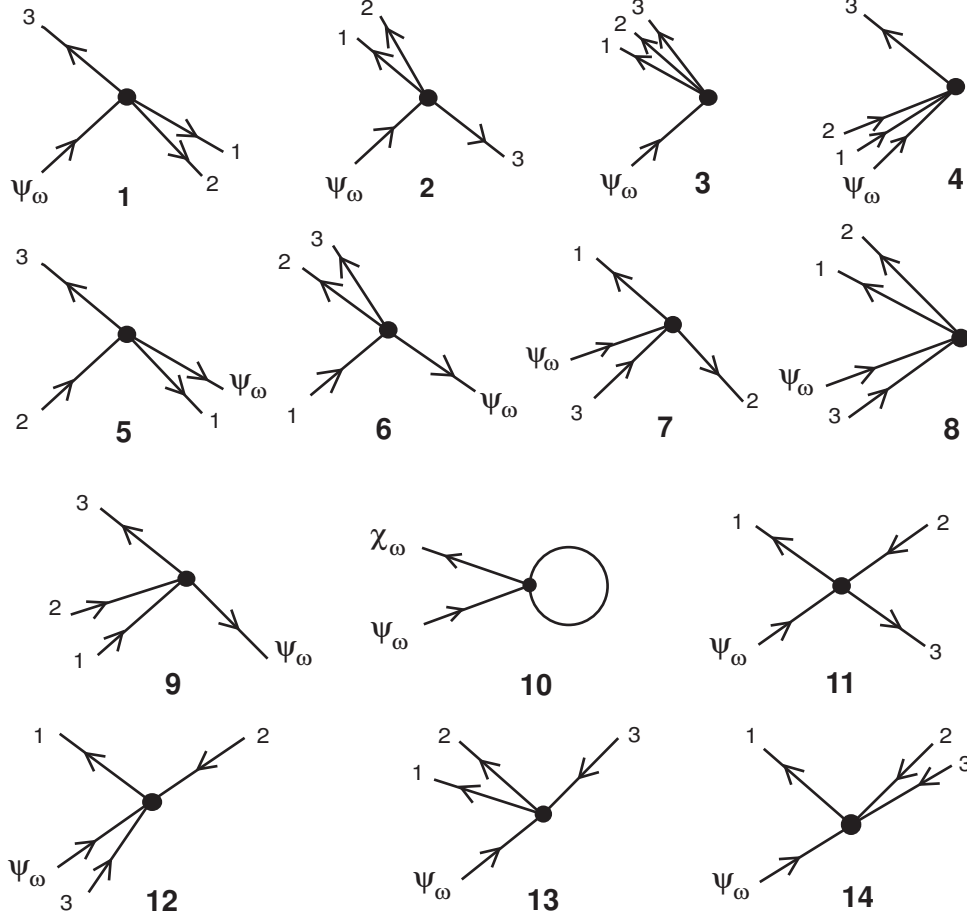


FIG. 2: Planar diagrams of the noncommutative theory that contributes to the flux of outgoing particles.

$$\begin{aligned}
& \times \left| \int dr dt \lambda [\psi_1^* \star \psi_2^* \cdot \psi_\omega \star \chi_3^* + \psi_1^* \star \psi_2^* \cdot \chi_3^* \star \psi_\omega + \psi_2^* \star \psi_1^* \cdot \psi_\omega \star \chi_3^* \right. \\
& \quad + \psi_2^* \star \psi_1^* \cdot \chi_3^* \star \psi_\omega + \psi_1^* \star \psi_\omega \cdot \psi_2^* \star \chi_3^* + \psi_1^* \star \psi_\omega \cdot \chi_3^* \star \psi_2^* \\
& \quad + \psi_\omega \star \psi_1^* \cdot \psi_2^* \star \chi_3^* + \psi_\omega \star \psi_1^* \cdot \chi_3^* \star \psi_2^* + \psi_1^* \star \chi_3^* \cdot \psi_2^* \star \psi_\omega \\
& \quad \left. + \psi_1^* \star \chi_3^* \cdot \psi_\omega \star \psi_2^* + \chi_3^* \star \psi_1^* \cdot \psi_2^* \star \psi_\omega + \chi_3^* \star \psi_1^* \cdot \psi_\omega \star \psi_2^*] \right|^2, \quad (33)
\end{aligned}$$

where the last factor encodes the noncommutative contribution of the diagram 1. If we sum only over states $|\alpha_2\rangle$ which have not energies $\omega, \omega_1, \omega_2$ or ω_3 and we use Eq. (17), we obtain

$$\frac{18}{\pi} \sum_{\omega_1, \omega_2} \sum_{\omega_3} \sum_{\alpha_1} p_{\alpha_1} \left(\left| \left\langle \beta \left| C_\omega^\dagger C_1 C_2 b_3 \right| \alpha_1 \right\rangle \right|^2 - \left| \left\langle \beta \left| C_1^\dagger C_2^\dagger b_3^\dagger C_\omega \right| \alpha_1 \right\rangle \right|^2 \right)$$

$$\times \left| \int dr dt \lambda F(\Theta) \cdot \psi_1^* \psi_2^* \psi_\omega \chi_3^* \right|^2, \quad (34)$$

where $F(\Theta)$ is given by Eq. (18). The thermal probability function p_{α_1} is given by

$$p_{\alpha_1} = \left(1 - e^{-\beta\omega}\right) \left(1 - e^{-\beta\omega_1}\right) \left(1 - e^{-\beta\omega_2}\right) \left(1 - e^{-\beta'\omega_3}\right) \exp \left[-\beta(k\omega + k_1\omega_1 + k_2\omega_2) - \beta'k_3\omega_3 \right]. \quad (35)$$

Finally, evaluating the matrix elements we get

$$\begin{aligned} & \frac{18}{\pi} \sum_{\omega_1, \omega_2} \sum_{\omega_3} \sum_{k, k_1, k_2, k_3} k(k_1 + 1)(k_2 + 1)(k_3 + 1) \exp \left[-\beta(k\omega + k_1\omega_1 + k_2\omega_2) - \beta'k_3\omega_3 \right] \\ & \times \left(e^{(\beta - \beta')\omega_3} - 1 \right) \left| \int dr dt \lambda F(\Theta) \cdot \psi_1^* \psi_2^* \psi_\omega \chi_3^* \right|^2. \end{aligned} \quad (36)$$

Now, performing the sums over k, k_i it yields

$$\frac{18}{\pi} g(\omega) \sum_{\omega_1, \omega_2} \sum_{\omega_3} \left(g(\omega_1) + 1 \right) \left(g(\omega_2) + 1 \right) \left(g'(\omega_3) + 1 \right) \left(e^{(\beta - \beta')\omega_3} - 1 \right) \mathcal{H}_\star(\omega, \omega_1, \omega_2, \omega_3), \quad (37)$$

where

$$g(\omega_i) = \left(e^{\beta\omega_i} - 1 \right)^{-1}, \quad g'(\omega_j) = \left(e^{\beta'\omega_j} - 1 \right)^{-1}, \quad (38)$$

$$\mathcal{H}_\star(\omega, \omega_1, \omega_2, \omega_3) = \left| \int dr dt \lambda F(\Theta) \cdot \psi_1^* \psi_2^* \psi_\omega \chi_3^* \right|^2, \quad (39)$$

with $i, j = 1, 2$. In the above expression every one of the sums in k and k_i are evaluated using the geometric series properly.

The usual (commutative) contribution [10, 11] to the flux of outgoing particles are regained from our previous equations by considering the expansion of \mathcal{H}_\star by using (28) and taking the limit $\Theta \rightarrow 0$. This yields precisely

$$\frac{9}{2\pi L^2} \frac{g(\omega)}{\omega} \sum_{\omega_1, \omega_2} \sum_{\omega_3} \frac{g(\omega_1) + 1}{\omega_1} \frac{g(\omega_2) + 1}{\omega_2} \frac{g'(\omega_3) + 1}{\omega_3} \left(e^{(\beta - \beta')\omega_3} - 1 \right) H(2\omega_3) \delta_{\omega, \omega_1 + \omega_2 + \omega_3}, \quad (40)$$

where we have used that $N = L^{-1/2}$ in Eq. (5) and we have defined

$$H(\omega) = \left| \int dr \lambda e^{i\omega r^*} \right|^2. \quad (41)$$

The integral $H(\omega)$ involving the radial dependence of the mode functions ψ 's and χ 's from the (commutative) interaction $\lambda\Phi^4$.

Now $H(\omega)$ can be evaluated just as in Refs. [10, 11]. Thus one can observe an IR divergent behavior in Eq. (40) when ω goes to zero if we permit than λ be different from zero (for

arbitrary values of r). To regularize this divergence we introduce a cut-off in the interaction at large distances. For the black hole model λ takes the following spatial dependence [10, 11].

$$\lambda = \lambda_{bh}, \text{ for } 2M < r < K \quad (42)$$

$$\lambda = 0, \text{ for } r > K, \quad (43)$$

where $K \gg 2M$. In order to see the asymptotic behavior of $H(\omega)$ we consider the expression for the square of the gamma function of the imaginary argument

$$\Gamma(iy)\Gamma(-iy) = |\Gamma(iy)|^2 = \frac{\pi}{y \sinh \pi y}, \quad (44)$$

where $\Gamma(z)$ is defined as usual by $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$. Now rewrite $H(\omega)$ in terms of the variable $\rho = r - 2M$ in Eq. (41) by using (44) to get

$$\begin{aligned} H(\omega) &\simeq \lambda_{bh}^2 \frac{\beta}{2\omega} \left[\sinh \frac{\beta\omega}{4} \right]^{-1}, \quad \omega \gg \frac{1}{K}, \\ H(\omega) &\simeq \lambda_{bh}^2 K^2, \quad \omega \ll \frac{1}{K}. \end{aligned} \quad (45)$$

From the asymptotic expansion one can see that $H(\omega)$ goes as $\frac{1}{\omega^2}$ in the limit when $\omega \rightarrow 0$. For large values of K the infrared (IR) divergences (at low frequencies) arise in two dimensions because the particle density does not decrease when the distance is increased. Thus the interaction among the various modes remains with the same intensity for arbitrarily large distances.

Now we go back to the evaluation of the contribution of $dF/d\omega|_2$ from our relevant diagram and its inverse. For the evaluation of the sums over the frequencies we take the continuous limit taking $L \rightarrow \infty$ and replacing $\sum_{\omega_i} \rightarrow \frac{L}{\pi} \int_{\pi/L}^\Lambda d\omega_i$, where Λ is an ultraviolet (UV) cut-off to regularize the possible UV divergences. We will come back later to this subject in the context of the noncommutative theory. It is worth mentioning that all diagrams in the figure (2), but the diagram 10 have very similar amplitudes which are given by the expression (40). The aim of the next subsection is the study of the effects of the noncommutativity in the flux of particles.

In the rest of this subsection we comment about the regularization of IR divergences in the flux $dF/d\omega|_2$. To begin with, one can note that for the commutative part of the diagram

1, the analysis [10, 11] showed that the IR divergences have the generic form

$$(A_1 L + B_1 \ln L + C_1)(A_2 L + B_2 \ln L + C_2), \quad (46)$$

where A_i , B_i , C_i are functions depending only of ω . To be more precise, each term of the Eq. (40) can be expanded in Taylor series and integrating out all terms, the result can be expressed as a power series of L . For instance, to obtain the leading term of Eq. (46) given by $A_1(\omega)A_2(\omega)L^2$, it is enough to take $\omega_1 \rightarrow 0$ and $\omega_2 \rightarrow 0$ in Eq. (40) as $\omega_1 = \omega_2 = \pi/L$ ($L \rightarrow \infty$). Thus we get

$$\frac{9}{2\pi^3} \left(\frac{g(\omega)}{\omega} \right) \left(\frac{g'(\omega) + 1}{\omega} \right) \left(e^{(\beta - \beta')\omega} - 1 \right) H(2\omega) \left(\frac{L}{\beta\pi} \right)^2.$$

The quadratic divergence L^2 is the more severe of the IR divergences presented in these systems.

Now we are in a position of discussing the behavior of the IR and UV divergences in the noncommutative theory. We will compute everything to the second order in Θ by using the diagram 1. All other diagrams (except the one-loop diagram) will have a similar behavior in the computation of $dF^*/d\omega|_2$.

B. The Computation of the Noncommutative Contribution to the Flux

From Eqs. (20) and (25), one can see that the noncommutative flux takes the form

$$\begin{aligned} \left. \frac{dF^*}{d\omega} \right|_2(\omega) = & \frac{1}{\pi} \sum_{\alpha} p_{\alpha} \left\{ \left\langle \alpha \left| S_1^{\dagger} [N_{\omega}, S_1] \right| \alpha \right\rangle + \Theta^2 \left\langle \alpha \left| S_1^{\dagger NC} [N_{\omega}, S_1] \right| \alpha \right\rangle \right. \\ & \left. + \Theta^2 \left\langle \alpha \left| S_1^{\dagger} [N_{\omega}, S_1^{NC}] \right| \alpha \right\rangle + \Theta^4 \left\langle \alpha \left| S_1^{\dagger NC} [N_{\omega}, S_1^{NC}] \right| \alpha \right\rangle \right\}. \end{aligned} \quad (47)$$

We note that the first term on the right hand side of the above equation is the commutative amplitude worked out originally by Leahy and Unruh in Ref. [10, 11].

Now we proceed to evaluate the noncommutative lowest correction. First notice that

$$\begin{aligned}
\left\langle \alpha \left| S_1^{\dagger NC} [N_\omega, S_1] \right| \alpha \right\rangle &= \left\langle \alpha \left| S_1^{\dagger NC} N_\omega S_1 \right| \alpha \right\rangle - \left\langle \alpha \left| S_1^{\dagger NC} S_1 N_\omega \right| \alpha \right\rangle \\
&= \left\langle \alpha \left| S_1^{\dagger NC} \right| \beta \right\rangle \left\langle \beta \left| N_\omega S_1 \right| \alpha \right\rangle - \left\langle \alpha \left| S_1^{\dagger NC} \right| \beta \right\rangle \left\langle \beta \left| S_1 N_\omega \right| \alpha \right\rangle \\
&= n_{\omega\beta} \left\langle \alpha \left| S_1^{\dagger NC} S_1 \right| \alpha \right\rangle - n_{\omega\alpha} \left\langle \alpha \left| S_1^{\dagger NC} S_1 \right| \alpha \right\rangle.
\end{aligned} \tag{48}$$

We also note that the following equation is fulfilled

$$\left\langle \alpha \left| S_1^\dagger [N_\omega, S_1^{NC}] \right| \alpha \right\rangle = n_{\omega\beta} \left\langle \alpha \left| S_1^\dagger S_1^{NC} \right| \alpha \right\rangle - n_{\omega\alpha} \left\langle \alpha \left| S_1^\dagger S_1^{NC} \right| \alpha \right\rangle, \tag{49}$$

The sum of both results leads to

$$\begin{aligned}
\left\langle \alpha \left| S_1^{\dagger NC} [N_\omega, S_1] \right| \alpha \right\rangle + \left\langle \alpha \left| S_1^\dagger [N_\omega, S_1^{NC}] \right| \alpha \right\rangle &= n_{\omega\beta} \left\langle \alpha \left| S_1^{\dagger NC} S_1 + S_1^\dagger S_1^{NC} \right| \alpha \right\rangle \\
&\quad - n_{\omega\alpha} \left\langle \alpha \left| S_1^{\dagger NC} S_1 + S_1^\dagger S_1^{NC} \right| \alpha \right\rangle.
\end{aligned} \tag{50}$$

Thus, to find the interference term one have to compute the matrix element of $S_1^{\dagger NC} S_1 + S_1^\dagger S_1^{NC}$. From Eq. (31) is easy to note that S_1^\dagger and S_1^{NC} have different matrix elements.

Now we analyze the noncommutative correction to the flux of outgoing particles $dF^*/d\omega$ which come from the second term on the left hand side of Eq. (50). In order to do that we will focus in the computation of a specific noncommutative diagram. To do such a computation we use Eqs. (26) and (31) to see the following

$$\begin{aligned}
[C_\omega^\dagger C_\omega, S_1^{NC}] &= \int dr dt \frac{i\lambda\Theta^2}{24(1-2M/r)^2} \sum_{\omega_1, \omega_2} \left\{ \omega_1^2 \omega_2^2 [C_\omega^\dagger C_\omega, (\Phi_I(x_1)\Phi_O(x_2) + \Phi_O(x_1)\Phi_I(x_2))\Phi_3\Phi_4] \right. \\
&\quad \left. + [C_\omega^\dagger C_\omega, \frac{-iM}{r^2} [\omega_1 (b_1^\dagger \chi_1^* + C_1 \psi_1) (\omega_2^2 \Phi(x_2)) + (1 \leftrightarrow 2)]] \Phi_3\Phi_4 \right\} + \text{permut.}, \\
&= \int dr dt \frac{i\lambda\Theta^2}{24(1-2M/r)^2} \sum_{\omega_1, \omega_2} \omega_1^2 \omega_2^2 \left\{ (\Phi_I(x_1)(\Phi_\omega^- - \Phi_\omega^+) + (\Phi_\omega^- - \Phi_\omega^+)\Phi_I(x_2))\Phi_3\Phi_4 \right. \\
&\quad \left. + (\Phi_I(x_1)\Phi_O(x_2) + \Phi_O(x_1)\Phi_I(x_2))(\Phi_3(\Phi_\omega^- - \Phi_\omega^+) + (\Phi_\omega^- - \Phi_\omega^+)\Phi_4) \right\} \\
&\quad + \Re[C_\omega^\dagger C_\omega, S_1^{NC}] + \text{permutations}
\end{aligned} \tag{51}$$

where we have used the notation and conventions of [11], i.e., $\Phi_\omega^- \equiv \psi_\omega^* C_\omega^\dagger$ and $\Phi_\omega^+ \equiv \psi_\omega C_\omega$. Once again, the word “permutations” stands that we have to use Eq. (27) in order to keep symmetry of the amplitude under permutations of external lines in the procedure. In the

last equation $\Re[C_\omega^\dagger C_\omega, S_1^{NC}]$ stands for the real part of the commutator. The reason for not writing explicitly it is because S_1 is antihermitian and when we add both contributions on the left hand side of Eq. (50), that term vanishes.

Now we proceed to evaluate the contributions of Eq. (51). For instance we take the term

$$\frac{i\Theta^2}{24} \int dr dt \lambda \left(1 - \frac{2M}{r}\right)^{-2} \sum_{\omega_1, \omega_2} \omega_1^2 \omega_2^2 \Phi_I(x_1) \Phi_O(x_2) \Phi_3(\Phi_\omega^- - \Phi_\omega^+) + \text{permut.}$$

with the rest of the terms giving similar expressions. Decomposing the fields in terms of their mode functions and using the diagrammatic rules described in Section V, we can see that some of the terms that appear in this product are: $C_\omega^\dagger b_1 C_2 C_3$ and $b_1^\dagger C_2^\dagger C_3^\dagger C_\omega$. Thus for the term selected previously in the noncommutative amplitude $\Theta^2 \langle \alpha | S_1^\dagger [N_\omega, S_1^{NC}] | \alpha \rangle$, the corresponding expression is given by

$$\left\langle \alpha \left| \frac{\Theta^2}{96} \int dr' dt' \lambda \Phi'^4 \int dr dt \lambda \left(1 - \frac{2M}{r}\right)^{-2} \sum_{\omega_1, \omega_2} \omega_1^2 \omega_2^2 \Phi_I(x_1) \Phi_O(x_2) \Phi_3(\Phi_\omega^- - \Phi_\omega^+) + \text{permut.} \right| \alpha \right\rangle. \quad (52)$$

The noncommutative vertex contained in the interaction term given by $[C_\omega^\dagger C_\omega, S_1^\star]$. Recalling the expansion $S_1^\star = S_1 + S_1^{NC}[\Theta^2] + \mathcal{O}[\Theta^4]$ noncommutative diagram 1 with the fat vertex, can be resolved into standard Leahy and Unruh diagrams [10, 11], but for higher derivative terms (see figure 3). In figure 3 we have written it at the second order in Θ .

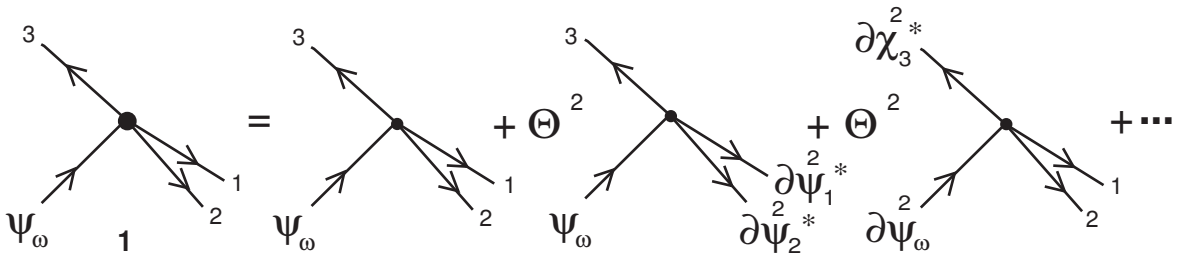


FIG. 3: Planar diagrams at second order in Θ , resulting from the resolution of the noncommutative (fat) vertex in terms of standard vertex and higher derivative terms.

We have seen in Eq. (37) that the noncommutativity is encoded in the Moyal products of the mode functions. This can be computed by using the noncommutative diagram 1,

interchanging the mode functions χ_3 and ψ_1 . Repeating the procedure described in previous sections for the term above we get the following expression for its amplitude

$$\frac{\Theta^2}{96\pi L^2} \frac{g(\omega)}{\omega} \sum_{\omega_1} \sum_{\omega_2} \sum_{\omega_3} \omega_1 \left(g(\omega_1)+1\right) \omega_2 \left(g(\omega_2)+1\right) \frac{g'(\omega_3)+1}{\omega_3} \left(e^{(\beta-\beta')\omega_3}-1\right) \tilde{H}(2\omega_3) \delta_{\omega, \omega_1+\omega_2+\omega_3}, \quad (53)$$

where $g(\omega_i)$ and $g'(\omega_i)$ were defined previously in Eq. (38) and $\tilde{H}(\omega)$ now have the following expression

$$\tilde{H}(\omega) = \int \lambda^2 \left(1 - \frac{2M}{r}\right)^{-2} \exp \left[i\omega(r^* - r^{*'}) \right] dr dr'. \quad (54)$$

It is worth mentioning that if the temperatures of ingoing and outgoing fluxes are equal, $\beta' = \beta$, then the noncommutative correction (53) vanishes. That means that, at least at the second order in Θ , even under the presence of the noncommutative interactions, both fluxes will be still thermal. Only for different temperatures $\beta' \neq \beta$, the noncommutative correction due the interaction to the outgoing flux still destroys the thermal nature of both interacting fluxes.

C. Some Implications of the Non-locality of the Noncommutative Theory

It is worth mentioning that near the event horizon the contribution of $\tilde{H}(\omega)$ to the noncommutative amplitude is not vanishing, which is a big difference with respect to the commutative case described in [10, 11]. To see this recall that the usual expression of the (commutative) outgoing flux $dF/d\omega|_2$ can be rewritten

$$\left. \frac{dF}{d\omega} \right|_2(\omega) = \frac{\lambda^2}{16\pi} \int dt dt' \int dr^* dr'^* \left(1 - \frac{2M}{r}\right) \left(1 - \frac{2M}{r'}\right) \sum_{\alpha} p_{\alpha} \langle \alpha | \Phi'^4 [C_{\omega}^{\dagger} C_{\omega}, \Phi^4] | \alpha \rangle \quad (55)$$

Equivalently we can say that the contribution of $H(\omega)$ in the limit $r \rightarrow 2M$ vanishes

$$\left| \int_{2M}^K dr \lim_{r \rightarrow 2M} \lambda e^{i\omega r^*} \right|^2 = \left| \int_{-\infty}^K dr^* \lim_{r \rightarrow 2M} \lambda \left(1 - \frac{2M}{r}\right) e^{i\omega r^*} \right|^2,$$

where we use the fact $dr/dr^* = (1 - 2M/r)$. This can be interpreted as the usual (commutative) interaction vanishes precisely in the event horizon [11]. This does not occur in the noncommutative case. Take for instance the contribution to the noncommutative amplitude

given by Eq. (52)

$$\frac{\Theta^2 \lambda^2}{96\pi} \int dt dt' \int dr^* dr'^* \left(1 - \frac{2M}{r'}\right) \left(1 - \frac{2M}{r}\right)^{-1} \left\langle \alpha \left| \Phi^4 \sum_{\omega_1, \omega_2} \omega_1^2 \omega_2^2 \Phi_I(x_1) \Phi_O(x_2) \Phi_3(\Phi_\omega^- - \Phi_\omega^+) \right| \alpha \right\rangle, \quad (56)$$

which doesn't vanish in the event horizon. Proceeding as in the previous commutative case we see that this fact can be explained if we look at the contribution to the noncommutative amplitude of $\tilde{H}(\omega)$ in the limit $r \rightarrow 2M$ and $r' \rightarrow 2M$ in Eq. (54) we have that

$$\int_{-\infty}^K \lim_{r' \rightarrow 2M} dr'^* \lambda \left(1 - \frac{2M}{r'}\right) e^{-i\omega r'^*} \left\{ \int_{-\infty}^K dr^* \lim_{r \rightarrow 2M} \lambda \left(1 - \frac{2M}{r}\right)^{-1} e^{i\omega r^*} \right\}, \quad (57)$$

which is in general non-vanishing. Moreover, the limit depends on the successions used for the approximation to the point $r = r' = 2M$ [33]. If we take iterating limits we will have that $\tilde{H}(\omega)$ diverges. A way of avoiding this divergence is to choose λ in a suitable way to eliminate the term $(1 - 2M/r)^{-1}$. However, such a procedure has not a clear physical justification. A similar situation has been revised in Ref. [10] for the case of flat space.

This allows to affirm that near and over the event horizon there exists a noncommutative contribution to the outgoing flux of particles coming from the matrix elements of the interference term. The only way of turning-off the effect of this noncommutative interaction precisely at the event horizon is to take off the noncommutative parameter being zero. An explanation of this behavior is because of the non-locality of the noncommutative interactions. Thus the noncommutative effects are present in a region of spacetime including the event horizon. Thus, one would expect a divergent behavior of the flux of particles justly in the event horizon due the noncommutative correction $\Theta^2 \langle \alpha | S_1^\dagger [N_\omega, S_1^{NC}] | \alpha \rangle$. This behavior come from the terms proportional to radial integrals $\tilde{H}(\omega)$ in the limit when $r \rightarrow 2M$.

D. The Structure of Divergences

In this subsection we discuss the behavior of the different divergences that appear in the evaluation of the noncommutative flux at diverse orders in Θ . All relevant diagrams described in figure 2 (with the exception of diagram 10) contain similar contributions to that given by Eq. (53). Then it is clear that the divergent terms coming from S_1 and S_1^{NC} have different behavior at the IR and that the main difference is that in S_1^{NC} the worst IR divergences of the type L^2 , are not present. Divergences of this type arise usually in the diagrams of the commutative theory.

Now we proceed to analyze the behavior of this contribution. We have seen that the commutative correction to the outgoing flux of particles [10, 11] have generic IR divergences even after renormalization. These divergences emerge from the expansion in Taylor series of the amplitude $\langle \alpha | S_1^\dagger [N_\omega, S_1] | \alpha \rangle$. In order to perform the computation in the noncommutative case it is convenient to recall some useful relations [11]:

$$\begin{aligned}
\frac{g(\omega_i)}{\omega_i} &= \frac{1}{\beta \omega_i^2} - \frac{1}{2\omega_i} + \dots \\
\frac{g(\omega_i) + 1}{\omega_i} &= \frac{1}{\beta \omega_i^2} + \frac{1}{2\omega_i} + \dots \\
\tilde{H}(2\omega - 2\omega_i) &= \tilde{H}(2\omega) - \omega_i \frac{d\tilde{H}}{d\omega}(2\omega) + \dots \\
\left(1 - e^{[(\beta - \beta')(\omega - \omega_i)]}\right) &= \left(1 - e^{[(\beta - \beta')\omega]}\right) + (\beta - \beta')e^{[(\beta - \beta')\omega]}\omega_i + \dots \\
\frac{g'(\omega - \omega_i) + 1}{\omega - \omega_i} &= \frac{g'(\omega) + 1}{\omega} \left[1 + \left(\frac{\beta'}{e^{\beta'\omega} - 1} + \frac{1}{\omega}\right)\omega_i + \dots\right], \tag{58}
\end{aligned}$$

with $i = 1, 2$. Substituting these expressions in Eq. (53) and taking $\omega_3 = \omega - \omega_1 - \omega_2$, we can see that we have a drastic difference than the commutative case. Now we don't have the generic quadratic leading term. Thus the first non-vanishing contribution in the noncommutative theory is given by

$$\frac{\Theta^2}{96\pi L^2} \frac{g(\omega)}{\omega} \frac{1}{\beta^2} \frac{g'(\omega) + 1}{\omega} \left(e^{(\beta - \beta')\omega} - 1\right) \tilde{H}(2\omega), \tag{59}$$

where we have used the fact $\lim_{\omega_i \rightarrow 0} \omega_i (g(\omega_i) + 1) = 1/\beta$. This last expression vanishes when we take the continuous limit (i.e $L \rightarrow \infty$). From this analysis we can see that the noncommutative correction to the flux due to the interference term $S_1^{\dagger NC} S_1$ will have not IR divergences. To compute the remaining divergences is necessary to take the continuous limit, changing the infinite sums over the frequencies ω_1 by integrals in expressions given in Eq. (58).

To find the different divergences which characterize the noncommutative contribution given by Eq. (53), we proceed in the following form:

- If we take $\omega_1 \neq \omega_2$. In this case we have that after taking the continuous limit the above expression takes the form

$$\frac{\Theta^2}{96\pi L^2} \left(\frac{L}{\pi}\right)^2 \frac{g(\omega)}{\omega} \int_{\pi/L}^{\Lambda} \int_{\pi/L}^{\Lambda} d\omega_1 d\omega_2 \omega_1 (g(\omega_1) + 1) \omega_2 (g(\omega_2) + 1) \frac{g'(\omega - \omega_1 - \omega_2) + 1}{\omega - \omega_1 - \omega_2}$$

$$\times \left(e^{(\beta-\beta')(\omega-\omega_1-\omega_2)} - 1 \right) \tilde{H}(2\omega - 2\omega_1 - 2\omega_2), \quad (60)$$

where we have used the energy conservation condition $\omega = \omega_1 + \omega_2 + \omega_3$. Substituting the Taylor expansion for each factor from Eq. (58) we can see that the first of these UV divergences takes the form

$$\frac{\Theta^2}{\pi^4} \frac{g(\omega)}{\omega} \left(\frac{\Lambda}{\beta} \right)^2 \left(\frac{g'(\omega) + 1}{\omega} \right) \left(e^{(\beta-\beta')\omega} - 1 \right) \tilde{H}(2\omega) = \frac{\Theta^2}{\pi^4} F(\omega) \Lambda^2, \quad (61)$$

where Λ is the UV cut-off introduced above. The other divergences are also UV.

- Now we consider the case with $\omega_1 = \omega_2$. Both outgoing particles have the same frequency and therefore the statistics is modified. To see that this is precisely the case we analyze the factor p_α :

$$p_\alpha = (1 - e^{-\beta\omega})(1 - e^{-\beta\omega_1})(1 - e^{-\beta'\omega_3}) \cdot \exp \left[-\beta(k\omega + k_1\omega_1) - \beta'k_3\omega_3 \right]. \quad (62)$$

Proceeding similarly to the above case, we can see that the contribution to the non-commutative flux of particles when $\omega_1 = \omega_2$ is given by

$$\frac{\Theta^2}{96\pi L^2} \frac{g(\omega)}{\omega} \sum_{\omega_1} \sum_{\omega_3} \left(\frac{g'(\omega_3) + 1}{\omega_3} \right) 2g^2(\omega_1) e^{2\beta\omega_1} \left(e^{(\beta-\beta')\omega_3} - 1 \right) \tilde{H}(2\omega_3) \delta_{\omega, \omega_1 + \omega_2 + \omega_3}. \quad (63)$$

Once again in the continuous limit we change the sums by integrals and use the energy conservation condition to remove one variable $\omega = 2\omega_1 + \omega_3$ and we obtain

$$\frac{\Theta^2}{96\pi L^2} \left(\frac{L}{\pi} \right) \frac{g(\omega)}{\omega} \int_{\pi/L}^{\Lambda} d\omega_3 \, 2g^2 \left(\omega - \frac{\omega_3}{2} \right) e^{2\beta(\omega-\omega_3)/2} \left(\frac{g'(\omega_3) + 1}{\omega_3} \right) \left(e^{(\beta-\beta')\omega_3} - 1 \right) \tilde{H}(2\omega_3). \quad (64)$$

This last expression does not posses IR nor UV divergences in the limit $\omega_3 \rightarrow 0$.

Then we can conclude that the terms proportional to Λ , Λ^2 , etc, coming from the interference, are the dominant in this case. These UV divergences will appear at different powers in Θ . For the correction of fourth order in Θ i.e. $\mathcal{O}(\Theta^4)$: $\Theta^4 \langle \alpha | S_1^{\dagger NC} [N_\omega, S_1^{NC}] | \alpha \rangle$ is possible to anticipate (using the Eqs. (58)) that there will be not IR divergences, only UV divergences.

Then we find a UV divergence of the form $\frac{\Theta^2}{\pi^4} F(\omega) \Lambda^2$. The rest of the divergences behaves as a power of the cut-off parameter Λ . The behavior of the divergences at higher order in Θ presents a UV behavior due to terms of the form

$$\int_{\frac{\pi}{L}}^{\Lambda} \omega_i^n d\omega_i = \frac{\Lambda^{n+1}}{n+1}, \quad (65)$$

in the corresponding amplitudes. The fact that IR do not arise here might be a consequence of the UV/IR at the second order in Θ . Finally UV divergences will be the only remaining divergences. This is expected due the noncommutative theories involve an infinite number of derivatives and therefore they are usually nonrenormalizable.

E. Noncommutative Correction in the Planar One-loop Diagram

Now we will consider the noncommutative correction in the computation of the diagram 10 of figure 2. This planar diagram is important because it have both IR and UV divergences. We are interested in the part of the noncommutative interaction Φ_\star^4 characterized by having one ingoing field Φ_I and one outgoing Φ_O given by Eqs. (6) and (7) respectively. Now we analyze the expansion of Φ_\star^4 in the following form

$$\Phi_\star^4 = \left(\Phi_I + \Phi_O \right)_\star^4 = \Phi_{I\star}^4 + 4\Phi_{I\star}^3 \star \Phi_O + 6\Phi_{I\star}^2 \star \Phi_{O\star}^2 + 4\Phi_I \star \Phi_{O\star}^3 + \Phi_{O\star}^4. \quad (66)$$

If we take the term $\Phi_I \star \Phi_{O\star}^3$ and make use of the properties of the Moyal product under the integral, then we have that

$$\begin{aligned} \Phi_I \star \Phi_{O\star}^3 &= \Phi_I \star \Phi_O (\Phi_{O\star})^2 \\ &= \Phi_I \star \Phi_O \left(C_\omega^2 \psi_{\omega\star}^2 + C_\omega C_\omega^\dagger \psi_\omega \star \psi_\omega^* + C_\omega^\dagger C_\omega \psi_\omega^* \star \psi_\omega + C_\omega^{\dagger 2} \psi_{\omega\star}^2 \right). \end{aligned} \quad (67)$$

If we take into account that the expectation values of the fields are given by $\langle \alpha | C_\omega^2 | \alpha \rangle = 0 = \langle \alpha | C_\omega^{\dagger 2} | \alpha \rangle$, then we have

$$\Phi_I \star \Phi_{O\star}^3 \rightarrow \Phi_I \star \Phi_O \left(C_\omega C_\omega^\dagger \psi_\omega \star \psi_\omega^* + C_\omega^\dagger C_\omega \psi_\omega^* \star \psi_\omega \right). \quad (68)$$

If we proceed similarly with $\Phi_{I\star}^3 \star \Phi_O$ one can see that

$$\Phi_{I\star}^3 \star \Phi_O \rightarrow \left(b_\omega b_\omega^\dagger \chi_\omega \star \chi_\omega^* + b_\omega^\dagger b_\omega \chi_\omega^* \star \chi_\omega \right) \Phi_I \star \Phi_O. \quad (69)$$

Then the part of Φ_\star^4 containing the term $\Phi_I \star \Phi_O$ is given by

$$\Phi_I \star \Phi_O = 12 \sum_{\omega_1} \left(\chi_{\omega_1}^* \star \chi_{\omega_1} + \psi_{\omega_1}^* \star \psi_{\omega_1} \right), \quad (70)$$

where the factor 12 represents all the combinations in which can be presented this state. Making use of the Moyal product, the last equation can be written as

$$\Phi_I \star \Phi_O = 12 \sum_{\omega_1} \left(\chi_{\omega_1}^* \chi_{\omega_1} + \psi_{\omega_1}^* \psi_{\omega_1} \right), \quad (71)$$

where we have used the fact $(\chi_{\omega_1}^* \star \chi_{\omega_1} + \psi_{\omega_1}^* \star \psi_{\omega_1}) \equiv (\chi_{\omega_1}^* \chi_{\omega_1} + \psi_{\omega_1}^* \psi_{\omega_1})$. This expression can be verified at each order of the noncommutativity parameter Θ . Obviously in order to take into account the symmetry of the total amplitude, we have to include the symmetry property (27).

That means that the noncommutative corrections of the diagram 10, come from the energy of the fields represented as external legs Φ_I, Φ_O . Thus Eq. (71) can be regarded as a representative which contains at the zeroth order in Θ , the results obtained previously [10, 11]

$$\Phi_I \Phi_O = 12 \sum_{\omega_1} \frac{1}{L\omega_1}. \quad (72)$$

In addition one can see that inserting this expression in the equation for the (commutative) flux $dF/d\omega|_2$ (see Eq. (40)) and taking the continuous limit $\sum_{\omega_1} \frac{1}{L\omega_1} \rightarrow \frac{L}{\pi} \int_{\pi/L}^{\Lambda} \frac{1}{L\omega_1} d\omega_1$, one have the contribution purely commutative of the diagram 10 to the outgoing flux

$$\frac{36\lambda^2}{\pi^3} f(\omega) \left(\ln \Lambda + \ln \frac{L}{\pi} \right)^2, \quad (73)$$

where

$$f(\omega) = - \left(\frac{g(\omega)}{\omega} \right) \left(\frac{g'(\omega) + 1}{\omega} \right) H(2\omega) \left[1 - \exp \left((\beta - \beta')\omega \right) \right]. \quad (74)$$

It is possible to add to the Hamiltonian interaction H_I of Eq. (16) a counterterm [10, 11] of the form $\int dr \lambda \delta m^2 \Phi^2$ such that the contribution to $dF/d\omega|_2$ is modified as follows

$$f'(\omega) \left[12 \frac{\delta m^2}{\pi^2} \left(\ln \Lambda + \ln \frac{L}{\pi} \right) + 4 \frac{(\delta m^2)^2}{\pi} \right], \quad (75)$$

where $f'(\omega)$ has the same form than in Eq. (74) (up to a constant factor). Then one can choice δm^2 such that all terms proportional to the result can be removed. This removes

the UV and IR quadratic divergences from the tree and one-loop diagrams. There are still terms of the form L , KL and $\ln L$. One can attempt to regularize the remaining IR and UV divergences.

If we take the diagram 10 and consider the contribution of the process containing the interaction of the fields $\Phi_I \star \Phi_O$

$$\Phi_I \star \Phi_O = 12 \sum_{\omega_1} \left(\chi_{\omega_1}^* \star \chi_{\omega_1} + \psi_{\omega_1}^* \star \psi_{\omega_1} \right) = \Phi_I \star \Phi_O = 12 \sum_{\omega_1} \frac{1}{L\omega_1}. \quad (76)$$

Noncommutative corrections only come from the term $\Phi_I \star \Phi_O$. Thus the noncommutativity does not affect the momentum of the internal loop and only affects the external legs of the diagram 10. This fact implies that the noncommutativity factorizes from the computation of the divergences at one loop and they will be not modified. This leads to planar diagrams and therefore the noncommutative theory will coincide with the commutative one. Of course the theory will have non-planar diagrams which will modify the nature of the divergences. As usual, there will be UV/IR mixing where UV divergences will be transformed into IR divergences [17].

F. Mass Renormalization

It is possible to add a mass term of the form $\lambda \delta m^2 \Phi_\star^2$ to the Hamiltonian to transform it into

$$\begin{aligned} H_I^\star &= \int dr \frac{\lambda}{4} \left(\Phi \star \Phi \star \Phi \star \Phi + \delta m^2 \Phi \star \Phi + \text{permutations} \right) \\ &= \int dr \frac{\lambda}{4} \left(\Phi \star \Phi \star \Phi \star \Phi + \delta m^2 \Phi^2 + \text{permutations} \right). \end{aligned} \quad (77)$$

Such a term does not affect the description of the noncommutative field theory. It propagates in the matrix S_1 and does not modify S_1^{NC} . When the mass term is introduced in this last equation, the flux of particles (commutative) $\langle \alpha | \Phi'^4 [C_\omega^\dagger C_\omega, \Phi^4] | \alpha \rangle$ is modified by the addition of the following terms:

$$4\delta m^2 \left(\left\langle \alpha \left| \Phi'^2 [C_\omega^\dagger C_\omega, \Phi^4] \right| \alpha \right\rangle + \left\langle \alpha \left| \Phi'^4 [C_\omega^\dagger C_\omega, \Phi^2] \right| \alpha \right\rangle \right) + (4\delta m^2)^2 \left\langle \alpha \left| \Phi'^4 [C_\omega^\dagger C_\omega, \Phi^2] \right| \alpha \right\rangle. \quad (78)$$

The first term already has been evaluated. In order to evaluate the remaining terms one proceeds similarly like the previous subsections. As in the commutative case, it is possible to

see that the terms proportional to the mass term have a contribution to the flux of particles as follows

$$g(\omega) \left(g'(\omega) + 1 \right) \left(\frac{H(2\omega)}{\omega^2} \right) \left(1 - \exp(\beta - \beta')\omega \right), \quad (79)$$

where the functions $g(\omega)$ and $g'(\omega)$ were previously defined and $H(2\omega)$ have the same structure as before. Is possible to choice δm^2 such that it can be removed from all terms of this type. However, this procedure only remove the more severe IR divergences (these are the quadratic ones L^2) and some of the UV divergences as well. The discussion about the behavior of the divergences present in the diagrams included would be repeated for the rest of the noncommutative diagrams coming for the interaction Hamiltonian (16) and (27).

VII. FINAL REMARKS

In the present paper we have studied the effect of a noncommutative interaction for the black hole model in two dimensions introduced in [10, 11]. The noncommutativity was implemented as a deformation of the product of mode functions ψ_ω and χ_ω , by introducing the Moyal product, then the interaction $\lambda\Phi^4$ is deformed into $\lambda\Phi \star \Phi \star \Phi \star \Phi$. Following the standard procedure of noncommutative field theory, we find the appropriate noncommutative generalization of the model and the set of planar diagrammatic rules introduced by Leahy and Unruh in Refs. [10, 11].

The noncommutative correction of the flux at the second order in λ in perturbation theory has been found. In order to obtain the explicit correction we compute it to the second order in the noncommutativity parameter Θ . To do that we compute the expansion in Θ from the interacting Hamiltonian (17) and then compute the S-matrix through S_1^{NC} . With this information the noncommutative correction to the flux of particles was estimated. This gives rise to a noncommutative correction even for planar diagrams (see Eq. (53)) to the result obtained by Leahy and Unruh [10, 11] (see Eq. (40)). Here we find that if the temperatures of ingoing and outgoing flux are equal, $\beta' = \beta$, then the noncommutative correction (53) vanishes. That implies that, at least at the second order in Θ , even under the presence of the noncommutative interactions, both fluxes will be still thermal. For different temperatures $\beta' \neq \beta$, the noncommutative correction due the interaction to the outgoing flux still destroys the thermal nature of both interacting fluxes. Thus the breaking of the nature of the thermal

flux remains under the noncommutative interaction. Another important result is that while the usual correction to the flux $\frac{dF}{d\omega}|_2$ due to the (commutative) interaction vanishes on the event horizon ($r = 2M$). In this paper we found that the noncommutative correction (56) is non-vanishing at the event horizon of the black hole (see Eq. (57)) and we interpret this as a non-local behavior of the noncommutative interaction. This is interpreted as a non-local effect which implies the presence of the scalar field in a region around the event horizon.

Divergences in the noncommutative correction term (53) were analyzed and the absence of IR divergences was found. Moreover, the only divergences are UV divergences of the form $\frac{\Theta^2}{\pi^4} F(\omega) \Lambda^2$. Higher order divergences (in Θ) behave as a positive powers of the UV cut-off parameter Λ . The fact that IR divergences do not arise here might be a manifestation of UV/IR mixing. Thus non-planar diagrams were not analyzed here and this deserves further analysis. Some of this work will be reported elsewhere.

Recently some work relating the gravitational anomaly and the Hawking radiation, appeared in the literature for a Schwarzschild black hole [34] and for a Reissner-Nordstrom black hole [35]. The extension to the rotating black hole has been considered recently in Refs. [36, 37]. The cancellation of the gravitational anomaly in Vaidya spacetime of arbitrary mass function was considered in Ref. [38]. It would be very interesting to study these works from the viewpoint of the results of this paper and those of [21]. Some of these ideas will be considered in a future work. The process of the black hole evaporation can be described with an analogy by using the propagation of sound waves in fluids [39]. It would be tempting to find a noncommutative deformation of this mechanical analogy and pursuing their consequences. In the present paper we have analyzed the Hamiltonian approach to field theory. It is worth studying the Lagrangian approach to quantum field theory in curved spaces in several dimensions, including the noncommutative nature of proper gravitational field. Finally, we would like to derive the model presented here, from string theory, in a similar sense of Ref. [27]. We will leave this for a future work.

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